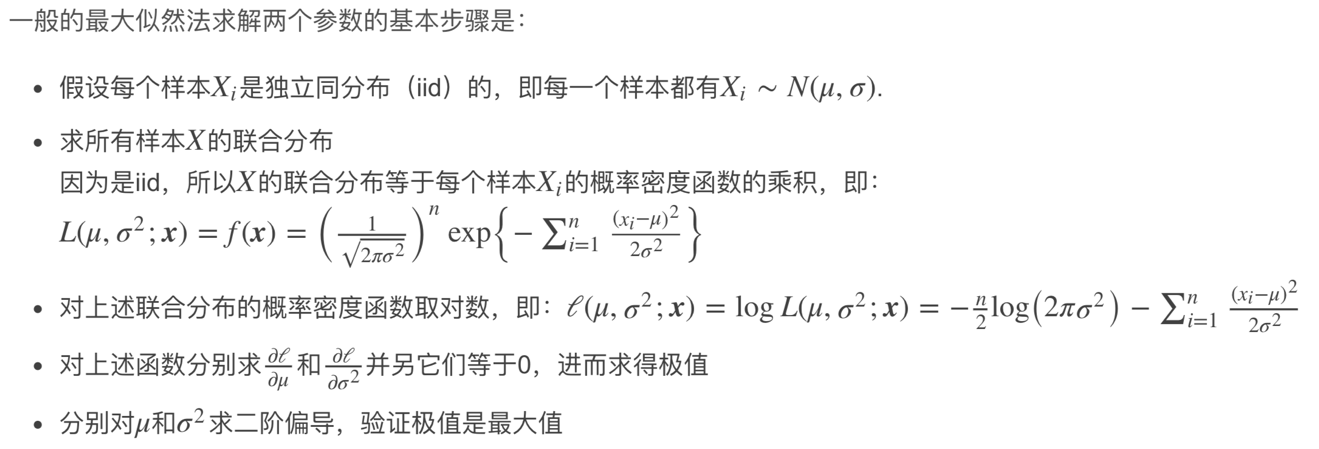
**Homework 4**

**Exercise 1:** Let us consider a univariate normal pdf with known variance. We propose to use a zero-mean normal pdf as a conjugate prior.

1. Find the following estimators: maximum likelihood, maximum *a posteriori*, and Bayesian estimator when square error loss function is used.

Square error loss:

L(θ̂ ,θ)=(θ̂ −θ)^2

MLE:

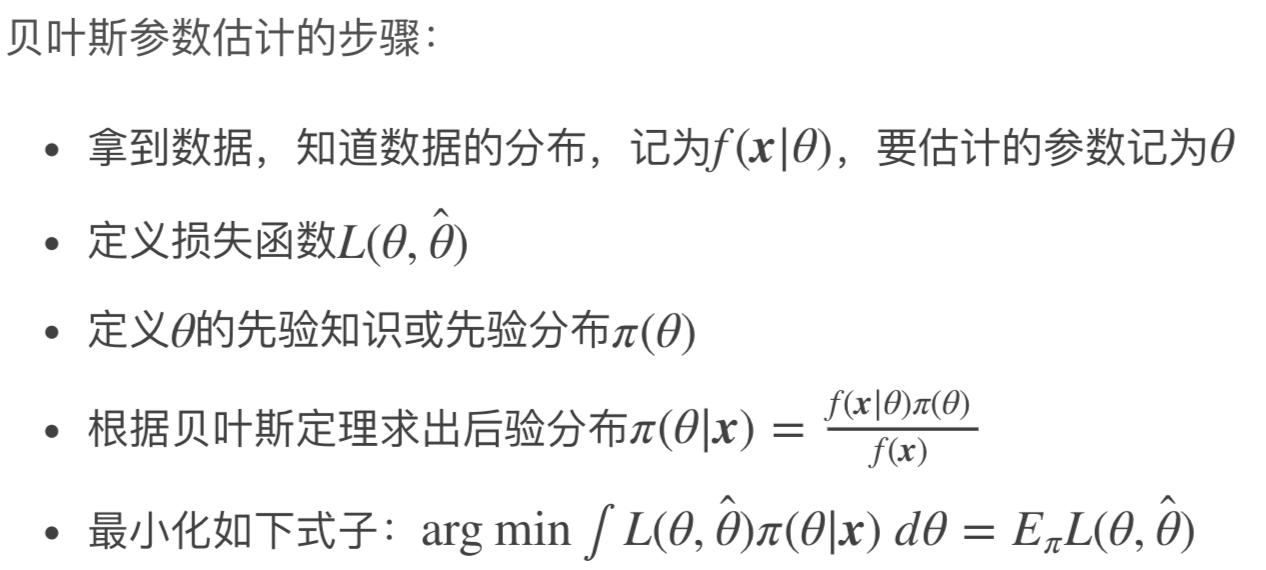
MAP:

π(θ|x)=f(x|θ)π(θ)/f(x)

f(x|θ)=MLE~N(miu\_hat,sigma)

π(θ)~N(0, sigma\_prime)

f(x)=

Bayes estimation: 

若损失函数是square error，那么当a等于θ在π(θ|x)上的期望时，贝叶斯风险最小

Square error下的贝叶斯估计就是后验分布的期望

2. By using real data, plot and compare the three estimators. You need to write a program in Python.

3. Now, we propose to transform the data used in 2. The transform is y=2x-1. Find the three estimators by using the transformed data.

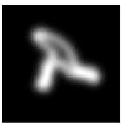
4. Compare the estimators you find in 1. and in 3.

**Exercise 2:** We would like to answer the following question. Given a binary image of black dots and a 2D shape, is this shape embedded in this image?



To answer the question we consider a parametric representation of the shape and we propose to assign a plausibility value for each black point (x,y) as follows:

σ is a scale parameter and the coordinates of a points belonging to the shape at the parameter s= . The resulting grey level image r(x,y) resembles to:



The issue is to identify the pixels belonging to the shape by using the classification. In the image below, we can see the pixels belonging to the shape.



It is expected that you:

1. Propose a Bayesian-based classification. You must provide the likelihood, the prior, and the posterior. The choice of pdfs should be argued.

2. Explain the algorithm you propose to use for the estimation of parameters.

3. Propose an algorithm for the classification. You must give all the steps of the algorithm.

**Deliverable:**

1. A report
2. Deadline: May 13th